NAG Toolbox for MATLAB

g05pc

1 Purpose

g05pc generates a realization of a multivariate time series from a vector autoregressive moving average (VARMA) model. The realization may be continued or a new realization generated at subsequent calls to this function.

2 Syntax

```
[x, iseed, r, ifail] = g05pc(mode, xmean, ip, phi, iq, theta, var, n,
igen, iseed, r, 'k', k)
```

3 Description

Let the vector $X_t = (x_{1t}, x_{2t}, \ldots, x_{kt})^{\mathrm{T}}$, denote a k dimensional time series which is assumed to follow a vector autoregressive moving average (VARMA) model of the form:

$$\begin{aligned} X_t - \mu = \quad & \phi_1(X_{t-1} - \mu) + \phi_2(X_{t-2} - \mu) + \cdots + \phi_p(X_{t-p} - \mu) + \\ & \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q} \end{aligned} \tag{1}$$

where $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{kt})^{\mathrm{T}}$, is a vector of k residual series assumed to be Normally distributed with zero mean and positive-definite covariance matrix Σ. The components of ϵ_t are assumed to be uncorrelated at non-simultaneous lags. The ϕ_i's and θ_j's are k by k matrices of parameters. $\{\phi_i\}$, for $i = 1, 2, \ldots, p$, are called the autoregressive (AR) parameter matrices, and $\{\theta_j\}$, for $j = 1, 2, \ldots, q$, the moving average (MA) parameter matrices. The parameters in the model are thus the p k by k ϕ-matrices, the q k by k θ-matrices, the mean vector μ and the residual error covariance matrix Σ. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & & . & & & & \\ . & & & . & & & \\ . & & & & . & & \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk} \quad \text{and} \quad B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & & . & & & & \\ . & & & . & & & \\ . & & & & . & & \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}$$

where I denotes the k by k identity matrix.

The model (1) must be both stationary and invertible. The model is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle and invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle.

For $k \geq 6$ the VARMA model (1) is recast into state space form and a realization of the state vector at time zero computed. For all other cases the function computes a realization of the pre-observed vectors $X_0, X_{-1}, \ldots, X_{1-p}$, $\epsilon_0, \epsilon_{-1}, \ldots, \epsilon_{1-q}$, from equation (1), see Shea 1988. This realization is then used to generate a sequence of successive time series observations. Note that special action is taken for pure MA models, that is for $p = 0$.

At your request a new realization of the time series may be generated with less computation using only the information saved in a reference vector from a previous call to g05pc. See the description of the parameter **mode** in Section 5 for details.

The function returns a realization of $X_1, X_2, \ldots, X_n$. On a successful exit, the recent history is updated and saved in the array **r** so that g05pc may be called again to generate a realization of $X_{n+1}, X_{n+2}, \ldots$,etc. See the description of the parameter **mode** in Section 5 for details.

Further computational details are given in Shea 1988. Note however that this function uses a spectral decomposition rather than a Cholesky factorization to generate the multivariate Normals. Although this method involves more multiplications than the Cholesky factorization method and is thus slightly slower it is more stable when faced with ill-conditioned covariance matrices. A method of assigning the AR and MA coefficient matrices so that the stationarity and invertibility conditions are satisfied is described in Barone 1987.

One of the initialization functions g05kb (for a repeatable sequence if computed sequentially) or g05kc (for a non-repeatable sequence) must be called prior to the first call to g05pc.

4 References

Barone P 1987 A method for generating independent realisations of a multivariate normal stationary and invertible ARMA(p,q) process *J. Time Ser. Anal.* **8** 125–130

Shea B L 1988 A note on the generation of independent realisations of a vector autoregressive moving average process *J. Time Ser. Anal.* **9** 403–410

5 Parameters

5.1 Compulsory Input Parameters

1: **mode – int32 scalar**

A code for selecting the operation to be performed by the function:

mode $= 0$ (start)

Set up reference vector and compute a realization of the recent history.

mode $= 1$ (continue)

Generate terms in the time series using reference vector set up in a prior call to g05pc.

mode $= 2$ (start and generate)

Combine the operations of **mode** $= 0$ and **mode** $= 1$.

mode $= 3$ (restart and generate)

A new realization of the recent history is computed using information stored in the reference vector, and the following sequence of time series values are generated.

If **mode** $= 1$ or 3, then you must ensure that the reference vector **r** and the values of **k**, **ip**, **iq**, **xmean**, **phi**, **theta**, **var** and **ldvar** have not been changed between calls to g05pc.

Constraint: $0 \leq$ **mode** ≤ 3.

2: **xmean(k) – double array**

μ, the vector of means of the multivariate time series.

3: **ip – int32 scalar**

p, the number of autoregressive parameter matrices.

Constraint: **ip** ≥ 0.

4: **phi($*$) – double array**

Note: the dimension of the array **phi** must be at least max(1, **ip** $\times$ **k** $\times$ **k**).

Contains the elements of the **ipk** $\times$ **k** autoregressive parameter matrices of the model, $\phi_1, \phi_2, \ldots, \phi_p$. If **phi** is considered as a three-dimensional array, dimensioned as **phi**(**k**, **k**, **ip**), then the (i,j)th element of ϕ_l would be stored in **phi**(i,j,l); that is, **phi**$((l-1) \times k \times k + (j-1) \times k + i)$ must be set equal to the (i,j)th element of ϕ_l, for $l = 1, 2, \ldots, p$ and $i,j = 1, 2, \ldots, k$.

Constraint: the first **ip** $\times$ **k** $\times$ **k** elements of **phi** must satisfy the stationarity condition.

5: **iq – int32 scalar**

q, the number of moving average parameter matrices.

Constraint: **iq** ≥ 0.

6: **theta**$(*)$ **– double array**

Note: the dimension of the array **theta** must be at least $\max(1, \mathbf{iq} \times \mathbf{k} \times \mathbf{k})$.

Contains the elements of the **iqk** $\times$ **k** moving average parameter matrices of the model, $\theta_1, \theta_2, \ldots, \theta_q$. If **theta** is considered as a three-dimensional array, dimensioned as **theta(k,k,iq)**, then the (i,j)th element of θ_l would be stored in **theta**(i,j,l); that is, **theta**$((l-1) \times k \times k + (j-1) \times k + i)$ must be set equal to the (i,j)th element of θ_l, , for $l = 1, 2, \ldots, q$ and $i,j = 1, 2, \ldots, k$.

7: **var(ldvar,k) – double array**

ldvar, the first dimension of the array, must be at least **k**.

var(i,j) must contain the (i,j)th element of Σ. Only the lower triangle is required.

Constraint: the elements of **var** must be such that Σ is positive-definite.

8: **n – int32 scalar**

n, the number of observations to be generated.

Constraint: **n** ≥ 0.

9: **igen – int32 scalar**

Must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialization by a prior call to g05kb or g05kc.

10: **iseed(4) – int32 array**

Contains values which define the current state of the selected generator.

11: **r(nr) – double array**

If **mode** $= 1$ or 3, then the array **r** as output from the previous call to g05pc must be input without any change to the first $m + (k+1)(k+2) + (m+1)(m+2)$ elements where $m = k \times \max(p, q)$ if $k \geq 6$ and $k(p+q)$ if $k < 6$.

If **mode** $= 0$ or 2, then the contents of **r** need not be set.

5.2 Optional Input Parameters

1: **k – int32 scalar**

Default: The dimension of the arrays **xmean**, **var**. (An error is raised if these dimensions are not equal.)

k, the dimension of the multivariate time series.

Constraint: **k** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

ldvar, nr, iwork, liwork

5.4 Output Parameters

1: **x(ldvar,$*$) – double array**

The first dimension of the array **x** must be at least **k**

The second dimension of the array must be at least $\max(1, \mathbf{n})$

$\mathbf{x}(i,t)$ will contain a realization of the ith component of $\mathbf{x}_t$, for $i = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, n$.

2: **iseed(4) – int32 array**

Contains updated values defining the new state of the selected generator.

3: **r(nr) – double array**

The first $m + (k+1)(k+2) + (m+1)(m+2)$ elements of the array **r** contain information required for any subsequent calls to the function with **mode** $= 1$ or 3; the rest of the array is used as workspace. See Section 8.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 0$.

ifail $= 2$

On entry, $\mathbf{k} < 1$.

ifail $= 3$

On entry, $\mathbf{ip} < 0$.

ifail $= 4$

On entry, $\mathbf{iq} < 0$.

ifail $= 5$

On entry, $\mathbf{ldvar} < \mathbf{k}$.

ifail $= 6$

On entry, $\mathbf{mode} < 0$ or $\mathbf{mode} > 3$.

ifail $= 7$

On entry, $\mathbf{liwork} < \mathbf{k} \times \max(\mathbf{ip}, \mathbf{iq})$.

ifail $= 8$

On entry, **nr** is too small.

ifail $= 9$

On entry, the covariance matrix Σ, as stored in **var**, is not positive-definite.

ifail $= 10$

This is an unlikely exit brought about by an excessive number of iterations being needed by the NAG Fortran Library function used to evaluate the eigenvalues of $A(\phi)$ or $B(\theta)$. If this error occurs please contact NAG.

ifail = 11

On entry, the autoregressive parameter matrices, as stored in **phi**, are such that the model is non-stationary.

ifail = 12

On entry, the moving average parameter matrices, as stored in **theta**, are such that the model is non-invertible.

ifail = 13

This is an unlikely exit brought about by an excessive number of iterations being needed by the NAG Fortran Library function used to evaluate the eigenvalues of the covariance matrix.

ifail = 14

g05pc has not been able to calculate all the required elements of the array **r**. This is likely to be because the AR parameters are very close to the boundary of the stationarity region.

ifail = 15

g05pc has not been able to calculate all the required elements of the array **r**. This is an unlikely exit brought about by an excessive number of iterations being needed by the NAG Fortran Library function used to evaluate eigenvalues to be stored in the array **r**. If this error occurs please contact NAG.

ifail = 16

Either **r** has been corrupted or the value of **ip** or **iq** is not the same as when **r** was set up in a previous call with **mode** = 0 or 2. To proceed, you should set **mode** to 0 or 2.

7 Accuracy

The accuracy is limited by the matrix computations performed, and this is dependent on the condition of the parameter and covariance matrices.

8 Further Comments

Note that, in reference to **ifail** = 12, g05pc will permit moving average parameters on the boundary of the invertibility region.

The elements of **r** contain amongst other information details of the spectral decompositions which are used to generate future multivariate Normals. Note that these eigenvectors may not be unique on different machines. For example the eigenvectors corresponding to multiple eigenvalues may be permuted. Although an effort is made to ensure that the eigenvectors have the same sign on all machines, differences in the signs may theoretically still occur.

The following table gives some examples of the required size of the array **r**, specified by the parameter **nr**, for $k = 1, 2$ or 3, and for various values of p and q.

p \ q	0	1	2	3
0	13 36 85	20 56 124	31 92 199	46 144 310
1	19 52 115	30 88 190	45 140 301	64 208 448

	35	50	69	92
2	136	188	256	340
	397	508	655	838
	57	76	99	126
3	268	336	420	520
	877	1024	1207	1426

Note that g13dx may be used to check whether a VARMA model is stationary and invertible.

The time taken depends on the values of p, q and especially n and k.

9 Example

```
mode = int32(2);
xmean = [5;
     9];
ip = int32(1);
phi = [0.8;
     0;
     0.07000000000000001;
     0.58];
iq = int32(0);
theta = [0];
var = [2.97, 0;
     0.64, 5.38];
n = int32(48);
igen = int32(1);
iseed = [int32(1762543);
     int32(9324783);
     int32(4234401);
     int32(742355)];
r = zeros(600, 1);
[x, iseedOut, rOut, ifail] = ...
    g05pc(mode, xmean, ip, phi, iq, theta, var, n, igen, iseed, r)
```

```
x =
  Columns 1 through 7
    5.7655    3.9830    0.4958   -0.7540   -1.7179   -2.7527   -0.5685
   14.7494   13.2462    8.8257    8.6140    8.7787    7.3103    5.9612
  Columns 8 through 14
    1.7186    2.3798    3.3195    3.1499    2.0514    1.3041    2.0792
    4.5940    5.3116    6.3535    6.8430    7.1364   10.6527    9.4157
  Columns 15 through 21
    2.9403    3.5389    3.8760    3.1787    7.2303    5.7914    6.2971
   10.0829   11.0119   10.7449    8.8071    4.3184    6.5094    7.5197
  Columns 22 through 28
    6.5077    7.9927    7.0411    5.5676    6.4044    6.2829    3.2301
    9.2047    9.8771    8.0817    8.4081   11.2183    9.6484    9.2601
  Columns 29 through 35
    0.9892   -0.3971   -0.3310    2.5261    1.6294    2.6930    3.0925
    6.6308   10.4070    7.7841    6.5094    5.1737    8.5939    7.8997
  Columns 36 through 42
    2.9668    5.1616    6.3311    8.1691   10.8006    9.1745    5.6266
    8.5752    8.3068    9.7257    5.3026    9.9886    9.9240   13.5973
  Columns 43 through 48
    7.6962    5.8435    1.5387    2.3767    1.3349   -0.1436
   16.1337   15.4641   11.8157    8.4282   11.2211    7.8694
iseedOut =
     6483542
    16686950
    12448015
     8596318
rOut =
     array elided
ifail =
```

```
         0
```